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SUMMARY

1. PURPOSE. To provide security and policy review on the document at Tab 1 prior to release to the public.

2. BACKGROUND.
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Title: New Special Relativity Effect Becomes Testable

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3. DISCUSSION.

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Tab
1. Paper

New Special Relativity Effect Becomes Testable

Kregg Arms and Mario Serna

February 20, 2014

Abstract

The equivalence-principle analog of the gravitational red shift in special relativity has never been measured. This red shift is the loss of synchronization associated with observes along a rigid beam being accelerated along a path preserving Born rigidity. We discuss some special conditions which simplify its experimental observation. Consider two initially synchronized clocks on the ends of a rigid rod that begins at rest and then accelerates along its length to a final velocity. Special relativity predicts that the two clocks initially synchronized will be shifted by an amount proportional to $\Delta\tau \approx Lv/c^2$. To the best of our knowledge, this special relativity prediction has never been observed, and experimental accuracy is just beginning to make this effect observable. We estimate the tolerance of the effect to experimental realities. If validated this new effect may one day aid in understanding and enhancing future ultra precision navigation systems.

Consider two clocks at different heights in a gravitational field. A gravitational redshift causes the clock deeper in the gravitational field to run slowly compared to the clock that is higher. Recently experiments have measured the red shift associated with less than one meter altitude difference [1].

In special relativity the equivalence principle analog of the gravitational red shift occurs when we take two clocks on opposite ends of a rigid rod and accelerate the rod along its long axis. We'll label the left clocks A and the right clock B. The two clocks are initially synchronized. After acceleration, they will no longer be synchronized. Rindler coordinates provide the special case where one can conveniently describe a rod of a fixed proper length (called Born rigidity) [2, 3] during a period of acceleration. Figure 1 shows this loss of synchronization on a space-time diagram. Light pulses are being emitted toward the center of a 100 meter rigid object at equal proper time intervals. At $t < 0$ one can see that the clocks A and B on each side of the rod are synchronized: $\tau_B - \tau_A \equiv \Delta\tau_{BA} = 0$. At $t = 0$ the rigid object begins to accelerate preserving Born rigidity. In figure 1 the rigid rod stops accelerating at $v = 0.4\,c$ which is around $c\,t \approx 30$ meters. Figure 1 also shows the surface of simultaneity when the rod's instantaneous velocity reaches $v = 0.4\,c$, and the rod ceases to accelerate. Before, during, and after the acceleration the proper-length of the rigid rod remains 100 meters [3]. After the acceleration the light pulses reaching the center point indicate that the two clocks on each end are no longer synchronized. Calculated using Rindler coordinates, the time difference is given by

$$c\Delta\tau_{BA} = L\tanh^{-1}(v) \approx L\,v \tag{1}$$

where the velocities are measured in a fraction of the speed of light. We show the derivation in the appendix. This is the time shift for the equivalence principle analog of the gravitational red shift.

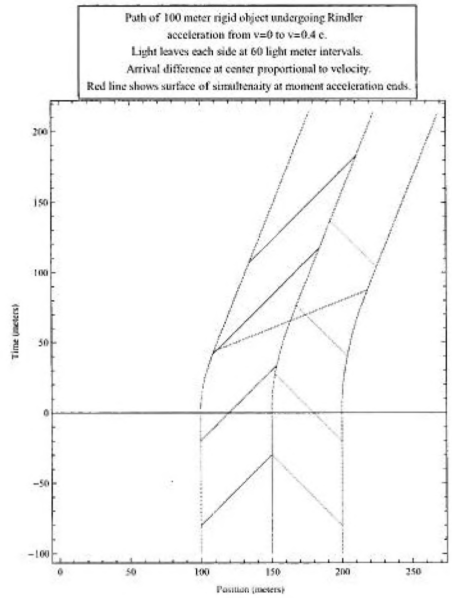

Figure 1: Time difference of pulses arriving from two clocks on opposite ends of a rigid rod indicates the velocity. Path of 100 meter rigid object undergoing Rindler acceleration from $v = 0$ to $v = 0.4c$. Light leaves each side at 60 light meter intervals. Arrival difference at center proportional to velocity. Red line shows surface of simultaneity at moment acceleration ends.

How big is the effect? One might imagine placing two clocks on opposite ends of an $L = 300$ meter cruise ship. When the ship goes from $v = 0$ meter/sec to $v = 10$ meter/sec to the right, then the clocks on the right side will be ahead of the left clock by about $\tau_B - \tau_A \approx 30 \times 10^{-15}$ seconds. The left clock, which had to undergo a larger acceleration will slow down relative to the right clock just as in the gravitational red-shift analog. Experimental techniques with this scale of accuracy have recently been demonstrated [1, 4]. There are efforts to make portable narrow-line-width lasers which would also be helpful for such a demonstration [5].

Our purpose here is not to specify the precise experimental approach, but rather to observe that this classic special relativity effect has never been observed and some elegant features help protect the effect against experimental complications. Experimental tests of special relativity are reviewed in Ref. [6]. The effects of linear velocity on time dilation is reviewed in Ref. [7]. In none of the past experimental tests were two clocks placed on ends of a rigid rod which was then linearly accelerated. The closest analog of the proposed test is the validation of the transverse Doppler effect in the Kündig experiment [8]. In the Kündig experiment a rigid disk is rotated with a source at the center and a detector at the radius [1].

Although the effect is easy to calculate with perfect Born rigidity in Rindler coordinates, the setup assumes noncausal accelerations. The system is noncausal because Born-rigidity requires the entire object to begin to accelerate non-causally at the same time $t = 0$. It also requires the entire object to non-causally stop accelerating along a surface of simultancity. We will now consider a more realistic scenario depicted in figure 2. This figure shows the space-time path for two clocks connected by a stiff rod. The left side of the rod is given a sudden push at $t = 0$ and changes velocity to v. The right side of the rod does not begin to move until the compression wave or sound signal reaches it. We will take the time required for the impulse on the left side to reach the right side to be L/v_s where L is the proper length of the rod and v_s is the speed of sound [3]. At the time a sound wave reaches the far end, it begins to move with a recovery speed v_r until

[1]The agreement of the Kündig experiment with special relativity is currently in dispute [9].

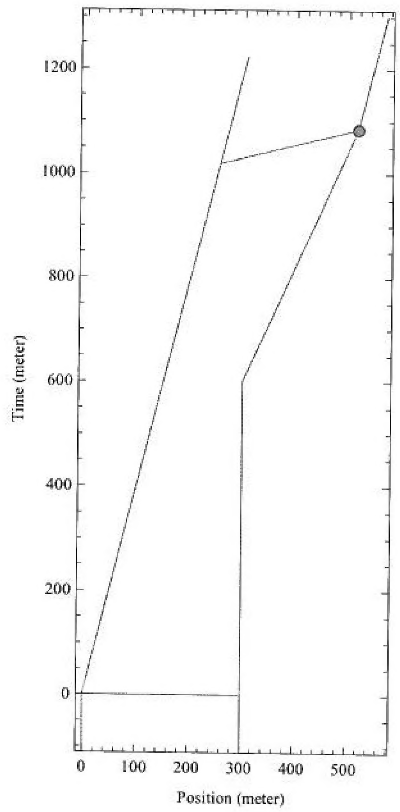

Figure 2: Space-time path for two clocks connected by a stiff rod. The left side of the rod receives an impulse at $t = 0$ and begins to move with a velocity v. The right side of the rod does not begin to move until the signal reaches it at the speed of sound v_s. At the time the sound wave reaches the far end, it begins to move with a recovery speed v_r until the rod reaches a new equilibrium length $L + \Delta L$.

the rod reaches a new equilibrium length $L + \Delta L$. For non-relativistic speeds of sound and recovery velocities, this impulse approximation gives the essential behavior. At the end of the transition when the entire rod now has proper length $L + \Delta L$ and is moving at a fixed speed v, the time difference between the two clocks A and B is now given as

$$\begin{aligned} c\Delta\tau_{BA} &= L\,\frac{(\gamma_v^{-1}-1)(v_r-(\gamma_r^{-1}+1)v_s)-v(\gamma_r^{-1}+v_r v_s-1)}{v_s(v-v_r)} \\ &\quad -\Delta L\,\frac{\left(\frac{1}{\gamma_v\gamma_r}+v\,v_r-1\right)}{v-v_r} \end{aligned} \tag{2}$$

where $\gamma_v = (1-v^2)^{-1/2}$ and $\gamma_r = (1-v_r^2)^{-1/2}$ and where all velocities are still expressed as a fraction of the speed of light. The derivation is found in the appendix. It is interesting to note that if $v_r = v_s = 1$ then $\tau_{BA} = 0$. To first order in the unitless small parameters v, v_r, v_s we have

$$c\Delta\tau_{BA} \approx L\,v\left(1-\frac{v_r}{2v_s}-\frac{\Delta L}{2\,L}(1+\frac{v_r}{v})\right)+\ldots. \tag{3}$$

Equation 3 has some important implications for the tolerance of an experiment to observe the induced time shift. First if we are dealing with something like a steel cruise ship, the speed of sound in Steel is $c\,v_s = 4300$ meters/sec. Therefore for typical boat speeds and accelerations $v_r/v_s << 1$ and $v/v_s << 1$ and both can be neglected. Next once the boat changes velocity, the length of the boat will change due to the drag force of the water. If $\frac{\Delta L}{L} << 1$ and $v_r/v \approx 1$, then the last terms are also unimportant. In the

setup described this means the time shift is dominated by $\Delta\tau_{BA} \approx L\,v$, A quick estimate of a steel boat suggest a 300 meter boat may be compressed by about 6 mm once the boat is moving at 10 m/s through the water [2]. Such a $\Delta L/L$ will not alter our prediction of the time shift induced by Born rigidity.

Although time dilation has been repeatedly measured in a gravitational field, the equivalence principle analog from linear acceleration has not yet been measured. We have shown that this effect is tolerant to experimental realities. The experimental accuracy required to measure this relativity phenomena is just now appearing. In a future era of ultraprecise clocks, such time differences will matter for two-way time transfers. This effect may one day have implications for precise navigation. After accounting for gravitational red shifts, one could in-principle determine the velocity relative to your starting velocity by measuring the time shift of two spatially separated clocks located on extreme ends of the boat, aircraft, or other vehicle.

Acknowledgements

We would like to thank Richard Cook, Scott Diddams, and Neil Ashby for comments on the manuscript.

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Appendix: Derivation of Key Equations

To derive eq. 1 we follow two paths of constant acceleration as described in Rindler coordinates representing different ends of a rigid rod starting at $t = \tau = 0$ and ending when the rigid rod has reached a velocity v. Rindler coordinate are given by $(x(\tau), ct(\tau)) = (c^2a^{-1}\cosh(a\tau/c), c^2a^{-1}\sinh(a\tau/c))$. An object moving along an accelerating path has a velocity $v(\tau) = \tanh(a\tau/c)$ where v is measured in m/sec. The left-side clock which starts at position $(x_A, 0)$ follows the trajectory with acceleration $a_L = c^2/x_A$. The clock on the right-side of the rigid rod of proper length L starts at position $(x_L + L, 0)$ follows the trajectory with acceleration $a_R = c^2/(x_A + L)$. When the left side reaches a velocity v, the proper time on that clock is $\tau_A = x_a/c\tanh^{-1}(v)$. When the right side reaches a velocity v, the proper time on that clock is $\tau_B = (x_a + L)/c\tanh^{-1}(v/c)$. The proper time difference is $\tau_B - \tau_a = L/c\tanh^{-1}(v/c)$.

To derive eq. 2 we parameterize the paths of the two clocks and then calculate the proper time elapsed between surfaces of simultaneity for the initial and final velocities. For clock A

$$(x_A(s),\ t_A(s)) = \begin{cases} (0,\ s) & \text{if } s \le 0 \\ (vs,\ s) & \text{if } s > 0 \end{cases}. \tag{4}$$

For clock B we have

$$(x_B(s),\ t_B(s)) = \begin{cases} (L,\ s) & \text{if } s \le \frac{L}{v_s} \\ (v_r\left(s - \frac{L}{v}\right) + L,\ s) & \text{if } \frac{L}{v_s} < s < s_R \\ (sv + \sqrt{1-(\frac{v}{c})^2}(L + \Delta L), s) & \text{if } s > s_R \end{cases} \tag{5}$$

where

$$s_R = \frac{Lv_s\left(\frac{\Delta L}{L}\sqrt{1-(\frac{v}{c})^2} + \sqrt{1-(\frac{v}{c})^2} - 1\right) + Lv_r}{v_s(v_r - v)}. \tag{6}$$

At $t = 0$ both A and B clocks are synchronized at 0. The time on each clock is calculated using

$$\tau = \int_0^{s_{\text{final}}} ds\sqrt{\left(\frac{dt}{ds}\right)^2 - \frac{1}{c^2}\left(\frac{dx}{ds}\right)^2}. \tag{7}$$

Using this the clock A reads

$$\tau_A = \int_0^{t'/\sqrt{1-(\frac{v}{c})^2}} ds\sqrt{1-(\frac{v}{c})^2} \tag{8}$$

where t' is the time coordinate in the moving frame of reference. After the object reaches a steady velocity, clock B reads

$$\tau_B = \int_0^{L/v_s} ds + \int_{L/v_s}^{s_R} ds\sqrt{1-(\frac{v_r}{c})^2} + \int_{s_R}^{(t'+\frac{v}{c^2}(L+\Delta L))/\sqrt{1-(\frac{v}{c})^2}} ds\sqrt{1-(\frac{v}{c})^2}. \tag{9}$$

The difference $\tau_B - \tau_A$ is evaluated using a computer algebra package is independent of t' and gives eq 2.